arXiv:hep-ph/0011015v2 8 Jan 2001

CCNY-HEP-00/6
hep-ph/0011015

Derivative expansion and gauge independence of the false vacuum decay rate in various gauges

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Abstract

In theories with radiative symmetry breaking, the calculation of the false vacuum decay rate requires the inclusion of higher-order terms in the derivative expansion of the effective action. I show here that, in the case of covariant gauges, the presence of infrared singularities forbids the consistent calculation by keeping the lowest-order terms. The situation is remedied, however, in the case of R_ξ gauges. Using the Nielsen identities I show that the final result is gauge independent for generic values of the gauge parameter v that are not anomalously small.

1 Introduction

The usual approach to the calculation of the false vacuum decay rate Γ yields an expression of the form [1]

$$\Gamma = A\, e^{-B} \tag{1}$$

where B is the classical action of the bounce solution of the Euclidean field equations and A is expression involving functional determinants which is generally of order one times a characteristic dimensionful quantity of the theory.

In theories with radiative symmetry breaking, however, this approach should be modified since the classical action may not even have a bounce solution. The obvious generalization would be to use the derivative expansion of the quantum Euclidean effective action

$$S_{\rm eff} = \int d^4x \left[V_{\rm eff}(\phi) + \frac{1}{2} Z(\phi)(\partial_\mu \phi)^2 \; + \cdots \right] \tag{2}$$

combined with a power series expansion in the coupling constants

$$V_{\rm eff}(\phi) = V_{e^4} + V_{e^6} + \cdots \tag{3}$$

$$Z(\phi) = 1 + Z_{e^2} + \cdots \tag{4}$$

where from now on we assume the general case of a gauge field theory with scalar coupling of order e^4, where the one-loop radiative corrections generate a symmetry-breaking minimum at $\phi = \sigma$, in addition to a symmetric minimum at $\phi = 0$.

One can then proceed to find a bounce solution to the equation

$$\Box \phi_b = \frac{\partial V_{e^4}}{\partial \phi} \tag{5}$$

and get a more complete expression for the false vacuum decay rate [2]:

$$\Gamma = A'\, e^{-(B_0 + B_1)} \tag{6}$$

where

$$B_0 = \int d^4x \left[V_{e^4}(\phi_b) + \frac{1}{2} (\partial_\mu \phi_b)^2 \right] \tag{7}$$

$$B_1 = \int d^4x \left[V_{e^6}(\phi_b) + \frac{1}{2} Z_{e^2}(\phi_b)(\partial_\mu \phi_b)^2 \right] \tag{8}$$

The bounce configuration, being a solution of Eq.(5), has a spatial extend of order $(e^2\sigma)^{-1}$, and accordingly, B_0 is of order e^{-4} and B_1 is of order e^{-2}.

The prefactor A' is the result of functional integrations, which includes higher-order terms in the effective action, and would be given by a dimensionful factor times a numerical factor which should be of order one, otherwise the approximations made in deriving Eq.(6) would break down.

The total expression for the vacuum decay rate, being a physical quantity, should be gauge independent. Since the leading order terms in the effective potential are gauge independent, it is obvious that the bounce solution and B_0 are gauge independent. The higher-order terms in the effective action, however, are gauge-dependent.

It was shown in [3] that in the case of scalar electrodynamics, with a Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu \Phi_1 - eA_\mu \Phi_2)^2 + \frac{1}{2}(\partial_\mu \Phi_2 + eA_\mu \Phi_1)^2 - V(\Phi) \tag{9}$$

where

$$V(\Phi) = \frac{1}{2} m^2 \Phi^2 + \frac{\lambda}{4!} \Phi^4 \tag{10}$$

and $\Phi \equiv (\Phi_1^2 + \Phi_2^2)^{1/2}$, in the general class of R_ξ gauges, with a gauge-fixing term

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}(\partial_\mu A^\mu + ev\Phi_2)^2 \tag{11}$$

the various terms in B_1 combine to give a ξ-independent result. A similar result regarding the ξ-independence of the bounce effective action was derived in [4] in the case of the $SU(2)$-Higgs model, also in the case of R_ξ gauges.

Here I will prove the v-independence of the false vacuum decay rate in the case of scalar QED in the class of R_ξ gauges, for generic (of order σ) values of the gauge parameter v. I will also describe how the calculation breaks down for values of v that are anomalously small.

The class of covariant gauges can be formally obtained from the class of R_ξ gauges by setting the parameter v to zero. One would therefore expect the results for the calculation of the false vacuum decay rate mentioned above for the R_ξ gauges to carry over to the case of the covariant gauges. I will show in this Paper that this is not the case, and the situation in covariant gauges

is quite different. In particular, I will show that higher-order terms in the derivative expansion of the effective action turn out to contribute at the same order of magnitude as the terms appearing in B_1, causing the entire formalism to break down. This is equivalent to saying that the prefactor A' in Eq. (6) grows larger, of the order of the exponential, making the approximations used in deriving Eq. (6) in [2] invalid. In other words, A' and e^{-B1} are of the same order of magnitude, are both gauge-dependent, and it is only the combined expression that should be gauge-independent. I do not have a proof of this fact in covariant gauges since I do not have a way of calculating the prefactor analytically. A possible way to investigate this would be along the lines of the calculation in [4].

In view of this fact, and the fact that covariant gauges can be formally obtained from R_ξ gauges by setting v to zero, it becomes interesting to check the gauge-independence of the result for B_1 in R_ξ gauges, now with respect to the other gauge parameter v. Using a set of Nielsen identities, similar to the ones used in [3], I will prove that the result is indeed v-independent, for generic values of the parameter v, generic meaning here of order σ. As v gets smaller, of order $e^2\sigma$ or less, higher-order diagrams start to contribute, and the whole calculation breaks down. It seems that v acts as a sort of infrared regulator that cures the infrared divergences of the covariant gauges. The price to pay is that it is an additional gauge parameter, and gauge-independence should be checked with respect to v as well.

Similar considerations and results of this paper would apply to any calculation of the false vacuum decay rate in theories where the radiative corrections are important. This includes theories with radiative symmetry breaking, and field theories at high temperature. Although the conclusions may vary for different models, the considerations for gauge independence via the Nielsen identities are an important check of the consistency of the calculation, and can be easily carried over to other models.

There are various calculations of the false vacuum decay rate in the framework of the $SU(2)$-Higgs model at finite temperature [5, 6, 7]. The analysis presented here can be extended to these models. The Nielsen identities are a special case of Ward identities that can be derived with sufficient generality [10] and can be verified algebraically in many cases [12]. In particular, the generalization to finite temperature and non-abelian theories is straightforward [10], although the verification may be more involved and the conclusions may be different (different models may have more or less severe divergences).

The organization of this paper is the following: In Sec. 2 I will describe the general method for checking the gauge independence of the false vacuum decay rate using the Nielsen identities. In Sec. 3 I show that the formula for the calculation of the decay rate breaks down in the case of covariant gauges because of higher-order terms in the effective action, and I show how this is reflected in the Nielsen identities. In Sec. 4 I show that in the case of R_ξ gauges the above problems do not exist for generic (of order σ) values of the gauge parameter v, and I show that the final value for B_1 is v-independent. Sec. 5 contains some concluding remarks.

2 The Nielsen identities

2.1 Basics

Consider a gauge theory with a set of fields denoted collectively by ϕ_i, with a classical action that is invariant under the infinitesimal gauge transformations

$$\delta_g \phi_i = \Delta_i \theta \tag{12}$$

where Δ_i are linear operators, and θ an abelian gauge parameter (the results can be extended to non-abelian gauge theories, but we will not consider that here).

After choosing a gauge-fixing function $F(\phi_i)$ and introducing Fadeev-Popov ghosts η and $\bar{\eta}$, one can proceed to a path-integral quantization and obtain the quantum effective action, $S_{eff}(\phi_i)$, which is the generating functional of the connected, one-particle irreducible Green's functions. The effective action, like the individual Green's functions, is not a physical quantity, and depends on the gauge-fixing condition.

One can derive a general formula that describes the change ΔS_{eff} of the effective action because of an infinitesimal change ΔF of the gauge-fixing condition [10]:

$$\Delta S_{\text{eff}} = i \int d^4x\, d^4y\, \frac{\delta S_{\text{eff}}}{\delta \phi_i(y)} \langle \Delta_i \eta(y)\, \bar{\eta}(x)\, \Delta F(x) \rangle_{\text{1PI}} \tag{13}$$

where the subscript 1PI indicates that only one-particle irreducible graphs are to be included. In particular, an infinitesimal change $d\xi$ in the gauge

parameter is equivalent to the choice $\Delta F = -(F/2\xi)d\xi$. Hence,

$$\xi\frac{\partial S_{\rm eff}}{\partial\xi} = \int d^4y \frac{\delta S_{\rm eff}}{\delta\phi_i(y)} H_i[\phi(z), y] \tag{14}$$

where

$$H_i[\phi(z), y] = -\frac{i}{2}\int d^4x \langle \Delta_i\eta(y)\,\bar\eta(x)\,F(x)\rangle_{\rm 1PI}\ . \tag{15}$$

Eq. (14) is the Nielsen identity [9], which can be used to prove the gauge independence of various physical quantities.

Our interest is to study the false vacuum decay rate given by Eq. (6). For simplicity we consider the case where the effective action depends on a single field $\phi(x)$, and make a derivative expansion of the single functional H that enters the Nielsen identity:

$$H[\phi(x), y] = C(\phi) + D(\phi)(\partial_\mu\phi)^2 + \cdots \tag{16}$$

Inserting this expansion together with the derivative expansion of the effective action into Eq. (14) we get

$$\begin{aligned} \xi\frac{\partial}{\partial\xi}\int d^4x \left[V_{\rm eff}(\phi) + \frac{1}{2}Z(\phi)(\partial_\mu\phi)^2 + \cdots\right] \\ = \int d^4x \Big[C(\phi) + D(\phi)(\partial_\mu\phi)^2 + \cdots\Big] \\ \left[\frac{\partial V_{\rm eff}}{\partial\phi} + \frac{1}{2}\frac{\partial Z}{\partial\phi}(\partial_\mu\phi)^2 - \partial_\mu\left[Z(\phi)\,\partial_\mu\phi\right] + \cdots\right]\ . \end{aligned} \tag{17}$$

Since this equation holds for arbitrary $\phi(x)$ the terms with equal number of derivatives must be equal. From the terms with no derivatives and from the terms with two derivatives we get respectively:

$$\xi\frac{\partial V_{\rm eff}}{\partial\xi} = C\frac{\partial V_{\rm eff}}{\partial\phi} \tag{18}$$

$$\xi\frac{\partial Z}{\partial\xi} = C\frac{\partial Z}{\partial\phi} + 2D\frac{\partial V_{\rm eff}}{\partial\phi} + 2Z\frac{\partial C}{\partial\phi}\ . \tag{19}$$

2.2 Scalar QED

Now we specialize to the case of scalar QED with gauge coupling e and scalar self-coupling λ of order e^4, that exhibits radiative symmetry breaking. We also limit ourselves to the class of R_ξ gauges, since the situation is different for covariant gauges, as we will show in the next section.

For the Lagrangian in Eq. (9) and the gauge-fixing in Eq. (11), the effective action can be obtained by making a shift $\Phi_1 \to \Phi_1 + \phi$ and dropping the linear terms [8]. The vertices can be read off from the remaining terms, and will depend on ϕ. The effective Φ_1, Φ_2 and A_μ propagators are

$$G_1(k) = \frac{i}{k^2 - m_1^2} \tag{20}$$

$$G_2(k) = \frac{i(k^2 - \xi e^2 \phi^2}{D(k)} \tag{21}$$

$$\begin{aligned} G_{\mu\nu}(k) &= G^T_{\mu\nu}(k) + G^L_{\mu\nu}(k) \\ &= -i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - e^2\phi^2} - \frac{i(\xi k^2 - e^2 v^2)}{D(k)}\frac{k_\mu k_\nu}{k^2} \end{aligned} \tag{22}$$

the mixed $A_\mu - \Phi_2$ propagator is

$$G_{\mu 2} = -\frac{e(\xi\phi + v)k_\mu}{D(k)} \tag{23}$$

for momentum flow from A_μ to Φ_2, and the ghost propagator is

$$G_g = \frac{i}{k^2 + e^2 v\phi} \tag{24}$$

In these expressions

$$D(k) = k^4 - k^2(m_2^2 - 2e^2 v\phi) + e^2\phi^2(e^2v^2 + \xi m_2^2) \tag{25}$$

and the various masses that appear are not the tree-level masses that one would have in theories without radiative symmetry breaking:

$$m_1^2(\phi) = m^2 + \frac{\lambda}{2}\phi^2 = V''(\phi) \tag{26}$$

$$m_2^2(\phi) = m^2 + \frac{\lambda}{6}\phi^2 = \frac{V'(\phi)}{\phi}\,. \tag{27}$$

Instead, in our case of radiative symmetry breaking, the fact that λ is of order e^4 implies that some multi-loop graphs are of the same order as graphs with fewer loops. In particular, the insertions of photon loops along a scalar propagator must be resummed. This can be done simply by replacing the above bare masses by dressed masses given by:

$$m_1^2(\phi) = V''_{e^4}(\phi) \tag{28}$$

$$m_2^2(\phi) = \frac{V'_{e^4}(\phi)}{\phi}\,. \tag{29}$$

These are the masses that should be considered in the above propagators.

The function $C(\phi)$ in the derivative expansion (16) can be obtained, to lowest order, from the two graphs with operator insertions shown in Fig. 1. Their sum is of order e^2 and given by

$$\begin{aligned} C_{e^2} &= -\frac{ie}{2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+e^2v\phi)\,D(k)}\Big[e(\xi\phi+v)\,k^2 - ev(k^2-\xi e^2\phi^2)\Big] \\ &= -\frac{ie^2\phi\xi}{2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{D(k)}\,. \end{aligned} \tag{30}$$

The next function $D(\phi)$ in the derivative expansion (16) is obtained from similar graphs with two external ϕ lines carrying momenta p and $-p$. Examination of these graphs shows that in the case of R_ξ gauges the lowest order graphs are of order one.

The expansion of the effective potential in power series in the coupling constant starts with the term of order e^4. The tree-level potential combines with the transverse photon loop to give

$$V_{e^4} = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 - \frac{3i}{2}\int \frac{d^4k}{(2\pi)^4}\ln(k^2-e^2\phi^2)\,. \tag{31}$$

The next term in the power series for the effective potential is of order e^6 and is given by

$$V_{e^6} = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\Big[\ln D(k) - 2\ln(k^2+e^2v\phi)\Big] \tag{32}$$

where the first term comes from the one loop graphs with Φ_2, longitudinal photon, or mixed scalar-photon propagators and the second term from the one loop ghost diagram.

The next term in the derivative expansion of the effective action (2) is given by

$$Z = -i\,\frac{\partial}{\partial p^2}\sum I_j\bigg|_{p^2=0}\,. \tag{33}$$

where $I_j(p^2)$ are the contributions of the diagrams shown in Fig. 2 with two external ϕ lines with momenta p and $-p$. In Fig. 2 we show only the diagrams of the lowest order, which is e^2.

2.3 Gauge independence

Now we can study the implications of the identities (18) and (19) when we consider the terms of the same order in the coupling constant. From (18), the terms of order e^4 give

$$\xi\frac{\partial V^{\text{eff}}_{e^4}}{\partial\xi} = 0\,. \tag{34}$$

which is the well-known result that the leading order term in the effective potential is gauge-independent [8], and it is obviously satisfied by (31).

The terms of order e^6 in (18) give

$$\xi\frac{\partial V^{\text{eff}}_{e^6}}{\partial\xi} = C_{e^2}\,\frac{\partial V^{\text{eff}}_{e^4}}{\partial\phi}\,, \tag{35}$$

which is the original form of the Nielsen identity [9] that was used to prove the gauge independence of the phenomenon of symmetry breaking and other physical quantities.

The terms of order e^2 in (19) give

$$\frac{1}{2}\xi\frac{\partial Z_{e^2}}{\partial\xi} = \frac{\partial C_{e^2}}{\partial\phi}\,. \tag{36}$$

This is a new form of a Nielsen identity that was derived in [3] for the case of R_ξ gauges.

Now we are ready to check the gauge-independence of the decay rate (6). Using the identities (35) and (36) we have

$$\xi\frac{\partial B_1}{\partial\xi} \;=\; \xi\frac{\partial}{\partial\xi}\int d^4x\left[V^{\text{eff}}_{e^6} + \frac{1}{2}Z_{e^2}(\partial_\mu\phi_b)^2\right]$$

$$
\begin{aligned}
&= \int d^4x \left[C_{e^2} \frac{\partial V_{e^4}^{\text{eff}}}{\partial \phi} + \frac{\partial C_{e^2}}{\partial \phi} (\partial_\mu \phi_b)^2 \right] \\
&= \int d^4x \left[C_{e^2} \frac{\partial V_{e^4}^{\text{eff}}}{\partial \phi} + (\partial_\mu C_{e^2})(\partial_\mu \phi_b) \right] \\
&= \int d^4x \, C_{e^2} \left[\frac{\partial V_{e^4}^{\text{eff}}}{\partial \phi} - \Box \phi_b \right] \qquad (37)
\end{aligned}
$$

and the last expression vanishes from the definition of the bounce.

The identities (35) and (36) where explicitly verified in [3] for the case of scaler QED in the class of R_ξ gauges, and the result for the decay rate was shown to be ξ-independent for generic values of the gauge parameter v. In the next section I will show that these identities break down in the case of covariant gauges, signaling the breakdown of the entire derivative expansion employed in the calculation of the decay rate from (6). Then in Section 4 I will show that, in spite of the infrared divergencies that plague the covariant gauges, the calculation in R_ξ gauges is well-defined with respect to the other gauge parameter, v, and the final result for the decay rate is v-independent. As mentioned before, this holds for generic values of the gauge parameter v, generic meaning of order σ.

3 The problems in covariant gauges

The situation in covariant gauges is quite different, and essentially the entire formalism described before for the calculation of the false vacuum decay rate breaks down [11]. Technically, the problems come from contributions of diagrams with longitudinal photon propagators. The function that appears in the denominator of the longitudinal photon propagator becomes

$$
D(k) = k^4 - k^2 m_2^2 + e^2 \phi^2 \xi m_2^2 \qquad (38)
$$

and the product of its roots is of order e^6 since m_2^2 is of order e^4. The product of the roots of the corresponding function in R_ξ gauges, (25), is of order e^4 for generic (of order σ) values of the gauge parameter v.

Consider a term of the next order in the derivative expansion of the bounce effective action, a term with four derivatives, of the form

$$
\int d^4x \, W(\phi) \, (\partial_\mu \phi_b)^2 \, (\partial_\mu \phi_b)^2 \ . \qquad (39)
$$

The function $W(\phi)$ receives contributions from graphs like the one in Fig. 3 with four external lines carrying momenta p and $-p$. It is easy to see that the ξ-dependent contributions of the terms with longitudinal photons in this graph are at least of order e^{-2}.

Remembering that the bounce has a spatial extent of order $(e^2\sigma)^{-1}$, we see that the contribution of (39) is of order e^{-2}, same as the factor B_1 in (8). This means that the calculation of the false vacuum decay rate from (6) is not self-consistent, as there are other contributions comparable to B_1, coming from higher order terms in the effective action. In fact, one can easily see through power-counting that all higher order terms have comparable contributions.

This situation is reflected in the Nielsen identities. Consider a graph of the form shown in Fig. 4, with external momenta p and $-p$, that contributes to the calculation of the quantity $D(\phi)$ appearing in (19). An easy calculation shows that the part of the graph coming from the longitudinal photons gives a contribution to $D(\phi)$ of order e^{-2}. Accordingly, instead of (36) we have

$$\frac{1}{2}\xi\frac{\partial Z_{e^2}}{\partial \xi} = \frac{\partial C_{e^2}}{\partial \phi} + D\frac{\partial V_{e^4}}{\partial \phi}\,. \tag{40}$$

The calculation of (37) does not hold, and the quantity B_1 is gauge dependent. The combination of e^{-B_1} and the terms of higher order in the derivative expansion, like (39), should be gauge-independent, but it does not seem possible to prove this perturbatively.

These problems do not arise in the case of R_ξ gauges, where the corresponding expressions for (39) and $D(\phi)$ are of order unity for values of v of order σ or larger. It is only for values of v of order $e^2\sigma$ or smaller that the calculation starts to break down. It seems that the gauge parameter v acts as a sort of infrared regulator, but of course the price to pay is that it is an additional arbitrary gauge parameter, and one has to check the gauge independence with respect to v as well. I will do that in the next section.

4 Gauge independence in R_ξ gauges

The ξ-independence of the decay rate in R_ξ gauges was proven in [3]. Here I will prove that the expression B_1 is also v-independent. Similar investigations of the gauge independence of other physical quantities in R_ξ gauges have been done in [12].

Starting from the most general form of the Nielsen identity, Eq. (13), one gets the change of the effective action with respect to the parameter v:

$$\frac{\partial S_{\text{eff}}}{\partial v} = \int d^4y \frac{\delta S_{\text{eff}}}{\delta \phi(y)} H^v[\phi(z), y] \tag{41}$$

where

$$H^v[\phi(z), y] = -ie^2 v \int d^4x \langle \Phi_2(y)\eta(y)\,\bar{\eta}(x)\Phi_2(x)\rangle_{1\text{PI}} \ . \tag{42}$$

The expansion

$$H^v[\phi(x), y] = C^v(\phi) + D^v(\phi)(\partial_\mu \phi)^2 + \cdots \tag{43}$$

starts with the quantity $C^v(\phi)$ which is given to lowest order by the diagram in Fig. 5, with the operator insertions of (42):

$$C^v_{e^2}(\phi) = -ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2 - \xi e^2 \phi^2}{D(k)(k^2 + e^2 v\phi)} \ . \tag{44}$$

The quantity $D^v(\phi)$ is again of order one, and proceeding in the same way as in Section 2, we get the Nielsen identities for the v-dependence of the effective action:

$$\frac{\partial V_{e^6}}{\partial v} = C^v_{e^2} \frac{\partial V_{e^4}}{\partial \phi} \,, \tag{45}$$

$$\frac{1}{2}\frac{\partial Z_{e^2}}{\partial v} = \frac{\partial C^v_{e^2}}{\partial \phi} \,. \tag{46}$$

Now one can proceed as in (37) to show that B_1 is v-independent.

I will now verify the Nielsen identities (45) and (46). The first identity can be easily verified algebraically, without explicit evaluation of the integrals. Starting from (32), and using the expression (25) we get

$$\begin{aligned} \frac{\partial V}{\partial v} &= -ie^2\phi \int \frac{d^4k}{(2\pi)^4} \frac{m_2^2(k^2 - \xi e^2\phi^2)}{D(k)(k^2 + e^2 v\phi)} \\ &= (\phi m_2^2)\, C^v(\phi) \ . \end{aligned} \tag{47}$$

Using (29) this proves the first identity (45).

The second identity (46) can also be verified algebraically, but because of the large number of terms it is easier to evaluate the relevant expressions explicitly. The quantity $C^v_{e^2}(\phi)$ is calculated from (44):

$$C^v_{e^2}(\phi) = -\frac{e^2}{(4\pi)^2}\left[\ln \frac{e^2 v\phi}{\mu^2} + \frac{\xi\phi}{2v} + \frac{1}{2}\right] \tag{48}$$

where the integral has been dimensionally regularized and minimally subtracted, and μ is the renormalization scale.

The quantity $Z_{e^2}(\phi)$ can be calculated from the diagrams of Fig. 2.

$$Z = -i \frac{\partial}{\partial p^2} \sum I_j \bigg|_{p^2=0} \ , \tag{49}$$

where $I_j(p^2)$ is the contribution of the diagram j with two external ϕ lines with momenta p and $-p$. We need the contributions of the diagrams that are proportional to p^2 and v-dependent. The sum of the first four is

$$\begin{aligned} & I_a + I_b + I_c + I_d = p^2 e^2 \int \frac{dk^4}{(2\pi)^4} \left[\frac{\xi}{D(k)} + \right. \\ & \left. + \frac{e^2 v^2}{D(k)^2} \left(\frac{3(e^2 v\phi)^2}{k^2 + e^2 v\phi} - \frac{3(e^2 v\phi)^2}{2k^2} - 5e^2 v\phi - 3k^2 \right) \right] = \\ & = -ip^2 \frac{e^2}{(4\pi)^2} \left(\xi \ln \frac{e^2 v\phi}{\mu^2} + \frac{25v}{12\phi} \right) \ . \end{aligned} \tag{50}$$

The ghost loop contribution is

$$\begin{aligned} I_e &= \frac{1}{2} p^2 e^6 v^3 \phi \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + e^2 v\phi)^4} \\ &= ip^2 \frac{e^2}{(4\pi)^2} \frac{v}{12\phi} \ . \end{aligned} \tag{51}$$

Combining (49), (50), (51) we get for the v-dependent part of $Z(\phi)$:

$$Z^v_{e^2} = -\frac{e^2}{(4\pi)^2} \left(\xi \ln \frac{e^2 v\phi}{\mu^2} + 2\frac{v}{\phi} \right) \ . \tag{52}$$

From the expressions (48), (52) we see that the second Nielsen identity, (46), is also satisfied.

5 Conlusion

In this paper I have shown that the calculation of the false vacuum decay rate via a derivative expansion of the effective action breaks down in the case

of theories with radiative symmetry breaking when the calculation is done in the class of covariant gauges.

I have also shown that these problems do not exist in the class of R_ξ gauges, for generic values of the gauge parameter v that are not anomalously small. In this case, the derivative expansion of the effective action can be used to calculate the correct, gauge-independent value of physical quantities.

Considerations of gauge independence are an important check of the consistency of the formalism. I showed that the above results are reflected in the Nielsen identities that describe the gauge dependence of the effective action. As a final check, I showed that the result for the false vacuum decay rate in R_ξ gauges is v-independent.

It is easy to formally generalize the calculations of this work and apply it to other models (non-abelian gauge theories or high temperature phase transitions) although the verification of the Nielsen identities in these cases may be more involved. The issue of gauge independence is still an important one, and considerations similar to this work may be helpful in order to clarify whether various calculations of physical quantities are self consistent or plagued by infrared divergences in different gauges.

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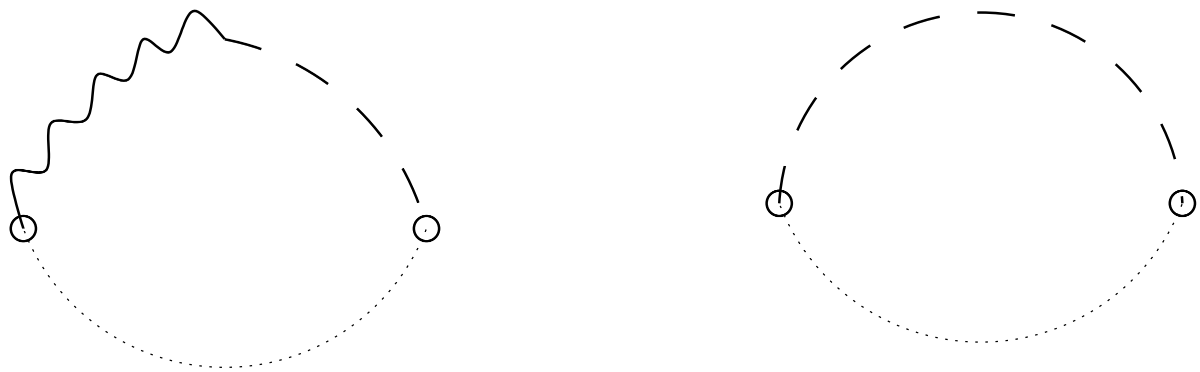

Figure 1: The two graphs that contribute to C_{e^2}. Photon, Φ_2, and ghost propagators are indicated by wiggly, long-dashed, and short-dashed lines, respectively.

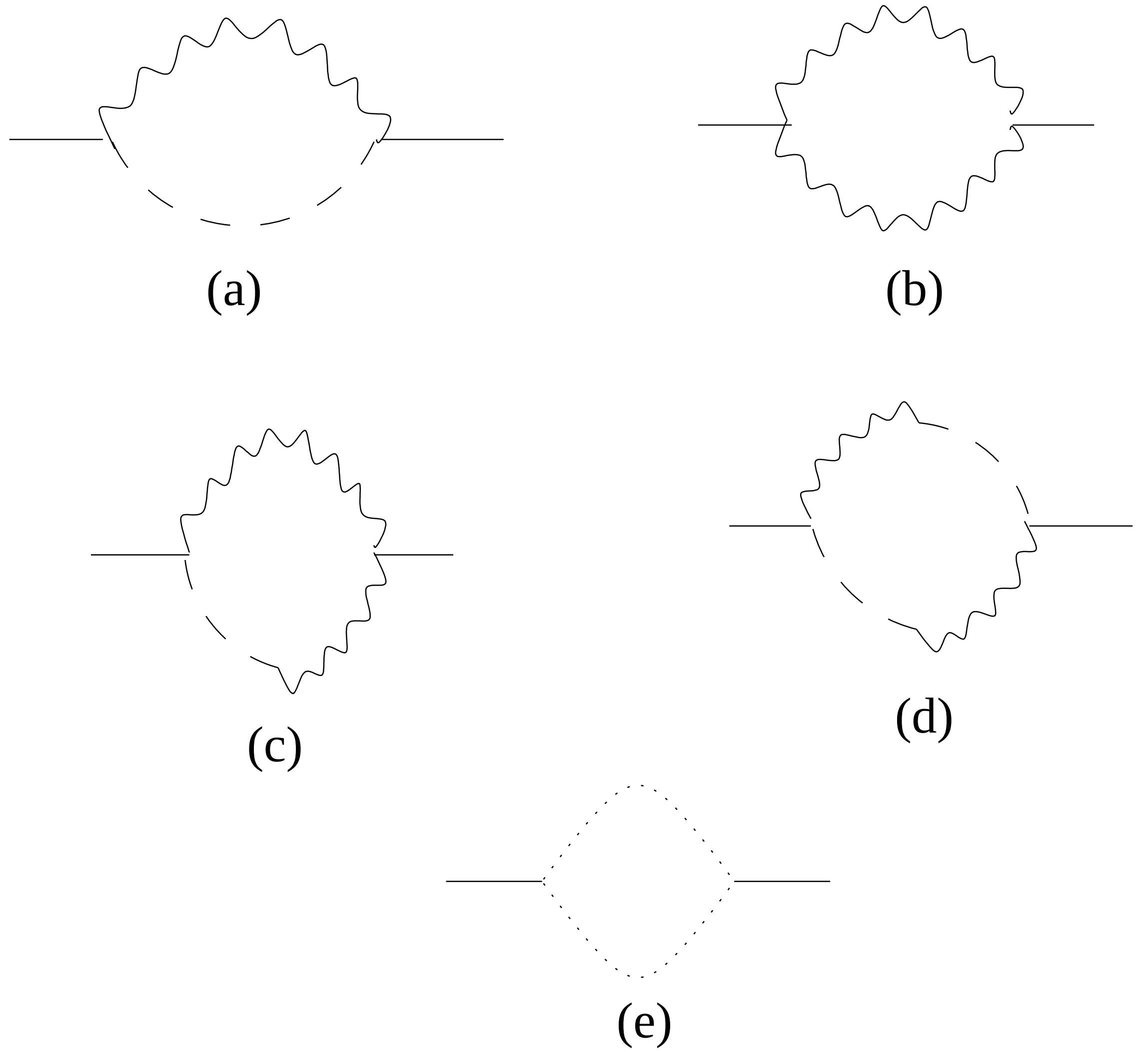

Figure 2: The graphs that contribute to Z_{e^2}.

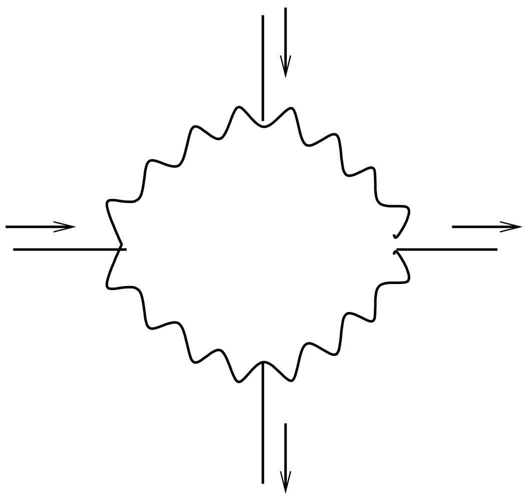

Figure 3: Example of a graph that signals the breakdown of the derivative expansion.

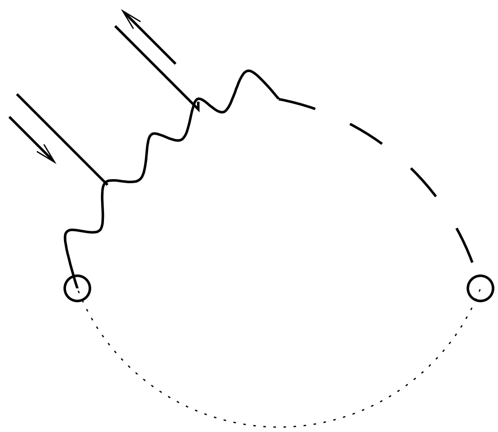

Figure 4: Example of a graph that contributes to the Nielsen identities.

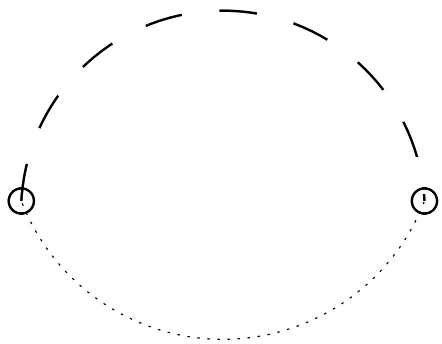

Figure 5: The graph that contributes to $C^v_{e^2}$.